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Characteristics of the Electric Arc

Electrical Engineering

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CHARACTERISTICS OF THE ELECTRIC ARC

BY

IPPEI KIYOHARA

B. S. State College of Washington, 1911

THESIS

Submitted in Partial Fulfillment of the Requirements for the

Degree of

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I HEREBY RECOMMEND THAT THE THESIS PREPARED UNDER MY SUPERVISION BY

Ippei Kiyohara

ENTITLED Characteristics of the Electric Arc.

BE ACCEPTED AS FULFILLING THIS PART OF THE REQUIREMENTS FOR THE

DEGREE OF Master of Science in Electrical Engineering

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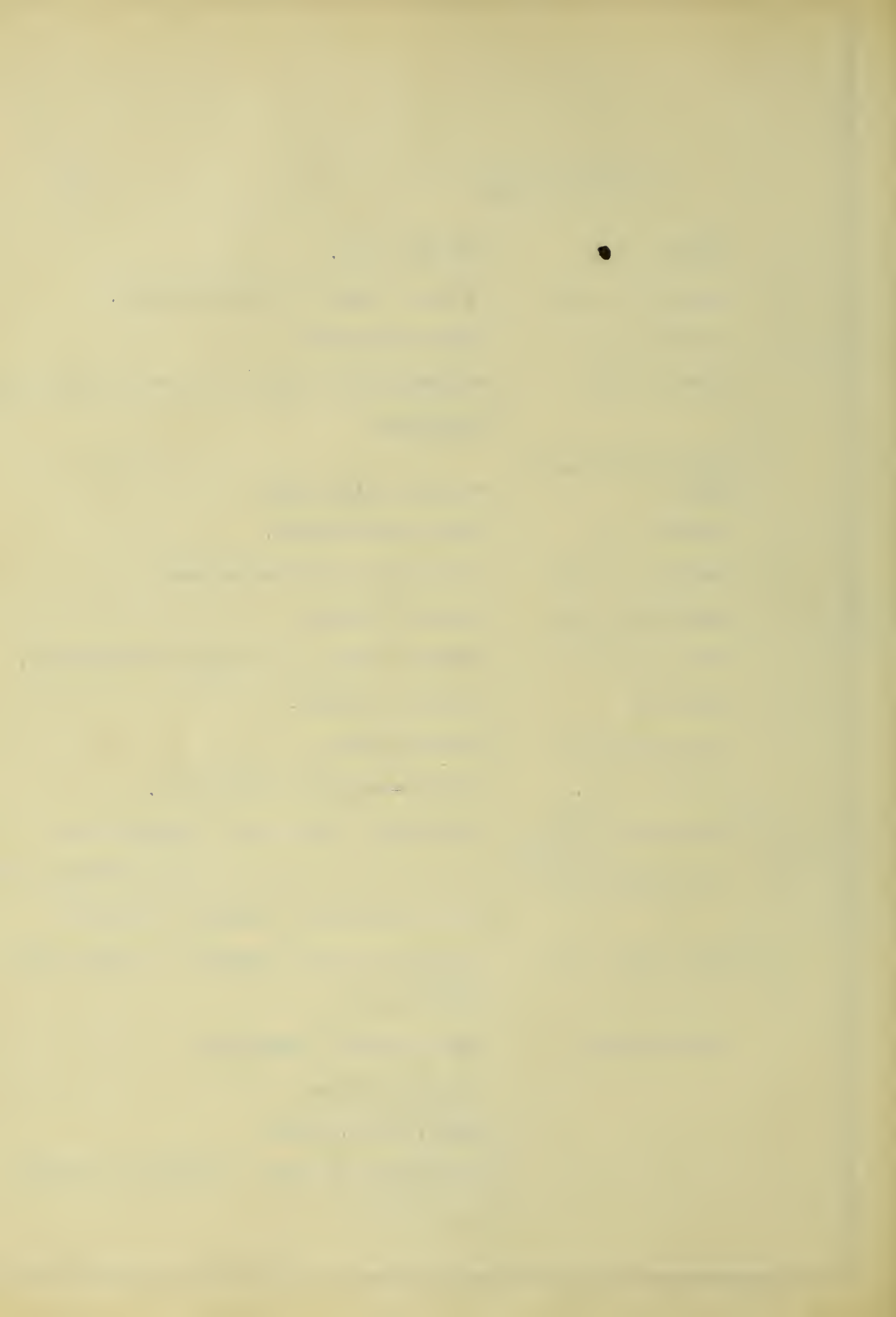
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II. Introduction.

The discoveries made during the last fifty years, and especially during the last ten years, concerning the radioactive substances and electric discharge through gases have thrown new light on the nature of electricity. The study of the electric arc or discharge in general is important not only because of practical applications to arc lamps, alternating mercury rectifiers etc., but because that is perhaps our only way of effectively getting at the nature of electricity itself. As long as the current is flowing through a conductor, its character can hardly be guessed. When, however, it passes through a gas, it reveals itself in brilliant light. It is through the study of such current, that is current conducted through gases, that we have arrived at the electron theory which gives an explanation of the nature of electricity and has succeeded the electrolytic theory.

Although a number of experimenters have investigated the nature of the electric arc and have tried to determine the law connecting the voltage across the arc, the current and the arc length, there is no one formula which can possibly be adopted as the appropriate law for any substance used for electrodes.

This is obviously due to the fact that most of these formulae are founded on experimental results without consideration of fundamental theory. Consequently these formulae differ because of the assumptions with which they are connected.

In the first part of this paper is given a brief description of the most general characteristics of the electric arc based on

experimental results. Also, several characteristic equations are presented which are compared and criticised.

In the second part, the most fundamental theory - the development of which is due to Faraday, Maxwell, Hertz, Lorentz and Larmor - is brought out in a concise form.

The third part is devoted to developing some mathematical expressions of the modern electronic phenomena based on both the theory and upon experimental results, which are more important and directly applicable to the proposed subject.

Though Mrs. Ayton, in her exhaustive study of carbon arc, suggests almost every possible explanation, her work is still far from satisfactory, mainly because she was unacquainted with the electron theory. Recently a few physicists have interest^{ed} themselves in the subject. The view of one of these, Prof. Pollock is discussed in detail in the latter of this paper.

In the last part, therefore, the writer attempts to give an explanation of the arc phenomena based on the modern electron theory.

III. The nature and the mechanism of an Electric Arc.

There are several ways to produce electric arc, viz; (1) raising the terminal potential difference high enough to cause an electric discharge and if the energy of this discharge is sufficiently large, a vapor stream will be formed, then the arc starts; (2) by supplying the conducting vapor stream from some other sources or raising the temperature of the gas between the terminals; (3) by bringing the electrodes into contact thus establishing the current in the circuit, and then separating them.

The last method is the usual and simplest way in starting an electric arc. Now, what would be the cause of formation of the arc? In general, it is explained by the aid of self induction. Owing to irregular surfaces of the solids, ~~the~~ all parts of the substances can not be separated at same time. In the course of separation, therefore, the small part which in contact still conducts the current and which has a greater resistance than the whole, will cause the greater heating. By this heat the substance is volatilized at the point of contact and in the next instant when there is no contact between them the small gap is full of the substance vapor and the arc is maintained even when the electrodes are separated further, although the vapor is changed, in certain extent according to the substance used as electrodes, into some different state due to the access of the cold air around.

Since, in arc conduction, Dr. Steinmetz writes in his book

"Radiation, Light and Illumination", the conductor is a stream of electrode vapor the color and the spectrum of the arc are those of the electrode substance and not of the gas, which fills the space in which the arc is produced and the nature of the gas in the space has no direct effect upon the arc. Nevertheless, the nature of the gas has some effect upon the potential difference of the arc. Dr. J.J. Thomson² declares that it is difficult to maintain good arc in pure hydrogen, due at least to more rapid convection of heat in this gas.

Mr. Arons³ has measured the potential difference required to produce an arc of 1.5 mm. long carrying a current of 4.5 amps. between terminals of different metals in air and pure nitrogen.

Terminal.	Potential difference.	
	air.	nit.
Ag.	21	—
Zn.	23	21
Cd.	25	21
Cu.	27	30
Fe.	29	20
Pt.	36	30
Al.	39	27
Pb.		18
Mg.		22

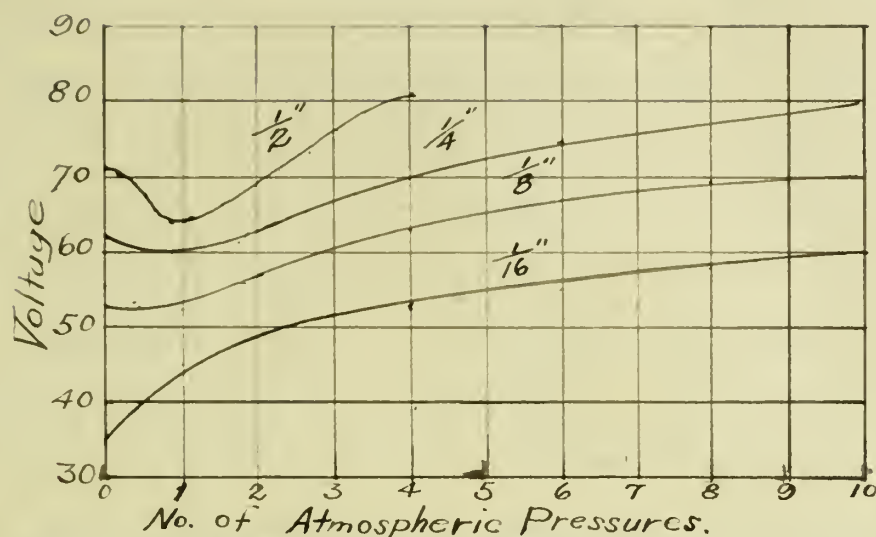
In the case of silver, he could not get any arc in pure nitrogen.

Messrs Duncan, Rowland and Todd have made a series of investigations on the effect of the pressure upon the potential

2. Conduction of Electricity through gases. J.J. Thomson.

3. Arons, Ann. der Phys. 1900.

difference in the arc and their results are graphically shown below:



*For four
Arc Lengths.*

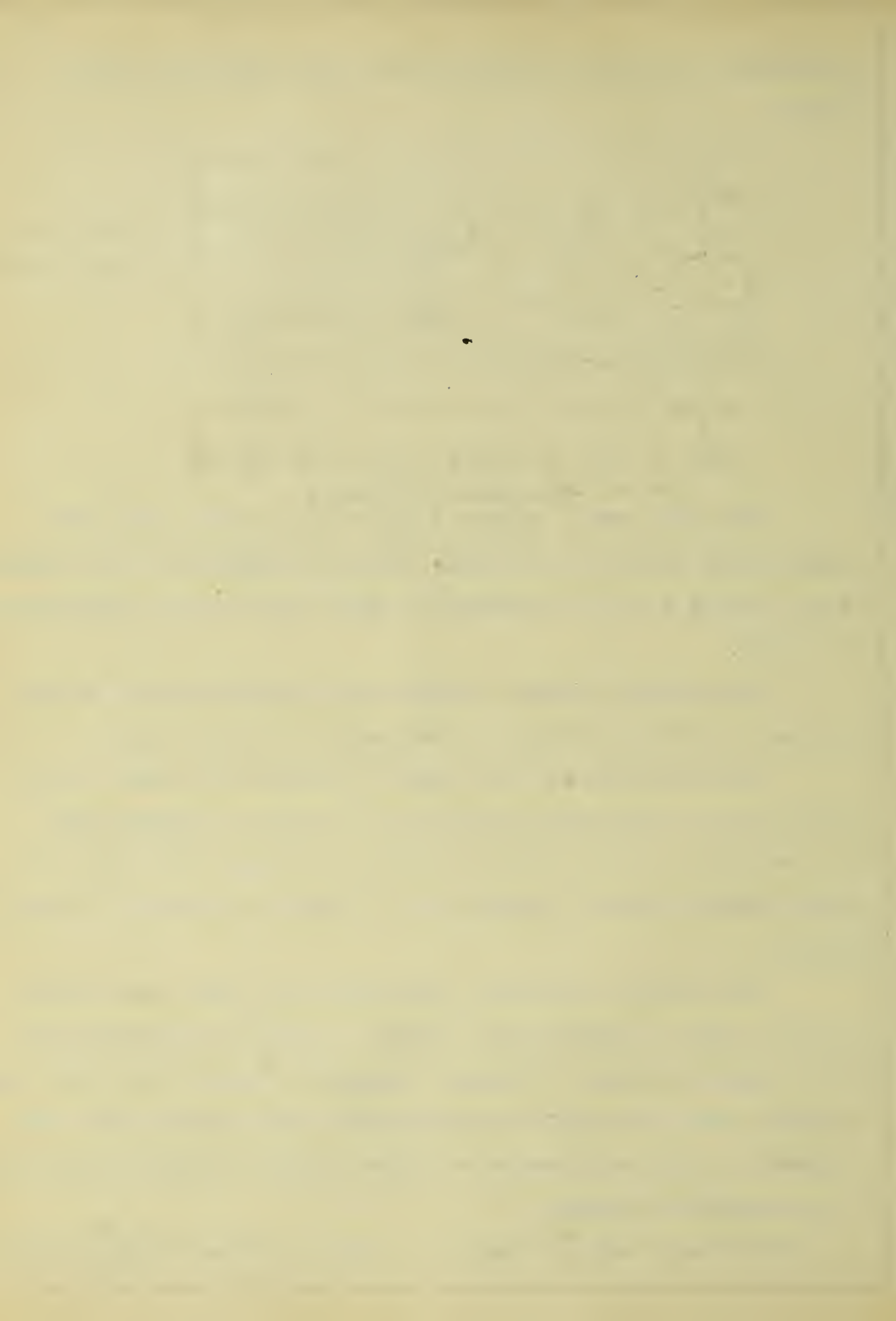
From this we will see that in short arc of 1/16 inch apart the potential difference increases with the pressure while in longer arcs there is a certain pressure at which the potential difference is lowest.

The pressure has some effect upon the luminousness of the arc due to the construction and expansion of the arc vapor.

The main sources of the light of a carbon arc light are the extremities of electrodes which are in a state of incandescence. The temperature of the positive crater, according to Violle², is about 3500°C while the extremity of the negative terminal is about 2700°C.

With metallic electrodes, generally, the light comes almost entirely from the luminous arc itself. The arc, however, is not of the same brightness throughout the entire length of it, but some brighter toward the positive terminal and very bright at the end adjacent to the same terminal and also brighter at the end adjacent to the negative terminal.

1. Electrician, Lond., Vol. 31 P.60. 2. Violle, Comptes Rendus, Vol. 65, 1892.



Resistance of Arc.

In general, the resistance of a normal arc is decreased in a greater rate than the current increases. It is seen in the results drawn from the experiment given later. The resistance of the arc and also the potential difference between electrode depend upon the current and the arc length as its relation is, in the later section, discussed and given in forms of equation.

Mrs Ayrton gives, in her book, some characteristics of its variation between carbon electrodes. As the matter of the fact, since the introduction of the electron theory (within recent few years) the attitude of the investigation and theory of the electric arc as well as the electric discharge through gases is considerably changed. Consequently the theories of those earlier investigators must be modified in certain extent, otherwise they are little value.

"Hissing Arc."

Beyond a certain ^{amount} of current, depending upon the length of arc, if the arc current is increased the voltage across the terminals suddenly falls about ten volts in the case of carbon electrodes. This current would remain almost constant in spite of further increase of the current and at this state the hissing sound is produced at the arc. Mrs. Ayrton has shown in her book that the extension of the crater up the side of the positive carbon causes the hissing due to the air which penetrates ^{the} surrounding vapor layer of the crater effecting the oxidation of the surface instead of volatilising, while in a silent arc the volatilising crater is protected by the vapor layer. With copper and brass the hissing arc can be found. In an iron arc, the hissing point can also be found analogous to the hissing point of the carbon arc, and

the bright spot at the positive terminal has tendency to travel around the surface of the terminal very rapidly.

Mr. W. G. Caddy (Physical Review, 1907,) could ^{not} find any hissing point with zinc and ^ualuminum electrodes.

Effect of a magnetic field on the arc.

The arc is deflected by a magnetic field in the same direction as a flexible wire would be if it carried a current flowing in the same direction as that through the arc. The resulting curved course is obviously a longer path and effect of a magnetic field on a potential difference is of the same character as an increase in length of the arc and just as it is possible to extinguish an arc by increasing its length, so the arc can be blown out by the application of a strong magnetic field.

Light efficiency of the Arc.

In the case of carbon arc, according to BLONDEL's¹ experiments, the greatest possible amount of light for a given power supplied with a certain current could be gotten by having the positive carbon as small as ^{Possible without} ~~it can be~~ but not ^e far of hissing, and negative carbon must also be smallest that will carry the current without burning it too fast with the most effective arc length, which, in certain limit, depending upon the given current and the nature of carbons that could be found by experiment.

Generally, in the carbon arc, differed from those of flame arcs, the useful light comes from the positive crater, the light efficiency is higher the shorter the arc.

The luminous efficiency depends also upon temperature of
1. BLONDEL; L. Eclairage Electrique, 1899.

the positive crater and thus depends, in turn, the nature of the carbon. Messrs W.H. DAMON and W. J. Anders², in their work have shown that both the teperature of the crater and radiant efficiency of the "C C forced carbon" were greater, almost in the same ratio, than those of the "Columbia cored carbon."

2. *Electrical World*; Sept., 1912.

IV. Relation between voltage across the arc, current in the arc, and the arclength.

There are several ways to determine this relation. The assumption is made, in most all treatments on this subject that the current and the length of arc are some function of the voltage across the arc. This assumption is reasonable and thus universally made.

$$E = f(I \ell)$$

With this as the general equation, we can derive, by the aid of experiments, the exact formula :

- (1) by determining the relation between E and I when ℓ is kept constant, then the relation between E and ℓ when I is kept constant, then combine the two;
- (2) by determining the relation between the arc resistance, E/I , and ℓ when I is kept constant, then relation between E/I and I when ℓ is kept constant and then combine the two;
- (3) by determining the relation between the power consumed in the arc, $E I$, and ℓ when I is kept constant, then the relation between $E I$ and I when ℓ is kept constant and then combine two;

and by several other combinations.

Mrs. Ayrton¹ took up the third method because the simplest, and she arrived at the general expression connecting the expended power, the current and the arc length, as

$$W = k_1 I + K_3 + \ell(k_2 I + k_4)$$

1. Mrs. Ayrton's Electric Arc.

from which the voltage equation is arrived as follows:

$$E = k_1 + k_2 l + \frac{k_3 + k_4 l}{I} \dots \dots \dots (A)$$

where k_1 , k_2 , k_3 and k_4 are constants depending on the substance, the shape, the size of the electrodes. l is distance between the electrodes and I is the current in the arc.

She gives the values of those constants from her experiments on the carbon electrodes; the diameter of the positive carbon was 11 mm. that of the negative carbon was 9 mm.

$$E = 38.88 + 2.074 l + \frac{11.66 + 10.54 l}{I}$$

The equation (A) is a rectangular hyperbola when l is kept constant having the asymptotes E and I .

Messrs A. Grau and F. Russ* give the results of their research on the relation between e.m.f., current and arc length of copper electrodes. The equation derived by them is

$$E = k_1 + k_2 l + \frac{k_3 l}{I}$$

which is without doubt a slight modification of Mrs. Ayrton's equation. The numerical values for those constants are for the copper arc; anode diameter = 18 mm. and cathode diameter = 12 mm., as follows:

$$k_1 = 55, \quad k_2 = 12.5 \quad \text{and} \quad k_3 = 90$$

$$\text{or } E = 55 + 12.5 l + 90 l / I .$$

Dr. Steinmetz** derived a formula which more or less differs from that of Mrs. Ayrton's. First he determined the relation between the voltage and the arc length. He ascertained the two

* Physi. Zeitscheschaft, Feb. 1908.

**Radiation, Light and Illumination, page 137.

parts of the voltage, one is the main part e_1 and the other is the magnetic arc voltage, as he called it, which is constant, that is independent of the arc length and the current in the arc. Also, he found an additional small imaginary arc length ℓ_1 which has the following relation

$$e_1 = k_1(\ell + \ell_1); \quad k_1 \text{ is constant.}$$

Then, from the voltage-current curves he made an assumption that the cubic hyperbola $e_1^2 = k_2^2$ or $e_1 = k_2/\sqrt{I}$, will express the close approximate relation of the voltage-current curve when the arc length is kept constant. Combining these two relations he obtained

$$e = e_0 + \frac{k(\ell + \ell_1)}{\sqrt{I}} \quad (s)$$

He gives the values for e_0 as 13 for mercury, 30 for magnetite, 36 for carbon; values of k as 123 for magnetite and 130 for carbon, when the arc measured in inches.

In comparing these two equations, we will see that both the fundamental bases of the derivation of the equations are almost same. Mrs. Ayrton's equation is entirely based on the experimental results and it is reasonable while in Dr. Steinmetz's equation, although it is also based on the experimental curves, the assumption of cubical hyperbola for the current-voltage curve is not always reasonable for in a long arc the curve does not follow this law.

As it is stated previously that there are several other ways to derive the connection between E, I and ℓ the results will be similar to either of those. Consequently no attempt is made to derive out any other characteristic equation.

V. The modern Electric Theory.

The modern view of electricity is found on a great number of elaborate investigations, to which contributions are made, not only by the electrostatic phenomena, but also by the phenomena of almost every branch of physics and chemistry. The modern explanation is similar to the older one-fluid theory. The electric fluid of the one-fluid theory can be supposed a crowd of small electrons or corpuscles, each having the same charge or quantity of negative electricity. These electrons always occur in exceedingly small particles and each has the same mass carrying the same quantity of electricity. The mass of an electron is about 8×10^{-28} and its charge is about 4.6×10^{-10} in electro-static units.

A substance in its neutral state contains a certain number of electrons, and these are just in such manner that two substances each in this state exert no electrical forces on one another. A substance becomes charged positively or negatively when the number of electrons it possesses is in less or excess of the number in the neutral state.

There is little information at present, about the positive electricity. The carrier of positive electricity or positive ions consist simply an atom or a particle removed an electron. Consequently its mass, even the smallest ever found (atom of hydrogen), is so large compared with the mass of electron as about 1700 times that of the latter.

VI. General Equations of motion of Electrons.

The electro theory based upon those fundamental mathematical treatments of Maxwell and Hertz which, are developed and applied in consideration of the problems of several branches of science by those noted people, as Lorentz, Larmor and very recently by J.J Thomson, Lodge, Rutherford, Jeans, Schott and others,

The practical application of mathematical theory is still in progress in wide range and there is yet a number of problems waiting for an explanation.

Thus, in the treatment in this subject it can not be useless but rather important to study and obtain, at least, the general idea of some essentials of the fundamental expressions, so as to lead one their correct applications for his investigations of the like subject.

Expressions of vectorial analysis of electronic motions.

As the vectorial ~~masses~~ ^{analysis} in treatment of mathematical physics is most effective way and advantageous in describing; it is skillfully adopted first by Hamilton and Lord Kelvin.

The basis of the present science founds on the view which is able to derive from the general philosophical idea that the assumption is constitute as discrete molecules aggregate existing in the aether. The aetherial character may be formulated by means of differential equations.

The electric fields belonging to material bodies furnish so

good a first approximation to the law of the electro dynamics of bodies in motion. The method of working out the dynamics of molecular systems is to formulate the problem in determination of the character of change of the configuration in the system, and so long as molecule are to be permanent, the system must be conservative; so that the principle of Least Action*, due to the MAUPERTUIS, is generally adopted.

First considering the dynamics of the free aether alone, and if a vector (X,Y,Z) represents the displacement at the point (x,y,z) , the fundamental well known equations for its kinetic energy, K , and potential energy, P are

$$K = 1/2 I \iiint (X^2 + Y^2 + Z^2) dx dy dz$$

$$= 1/2 I \int (X^2 + Y^2 + Z^2) d\tau \dots\dots\dots (1)$$

$$P = 1/2 M \iiint (f^2 + g^2 + h^2) dx dy dz$$

$$= 1/2 M \int (f^2 + g^2 + h^2) d\tau \dots\dots\dots (2)$$

where I = a constant of inertia,

M = a constant, modulus of elasticity,

d = element of volume,

and (f,g,h) is a solenoidal vector which has relation to (X,Y,Z) in the following manner:

$$\text{Curl } (f,g,h) = \begin{vmatrix} i & j & k \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ X & Y & Z \end{vmatrix} = \frac{dZ}{dy} - \frac{dY}{dz}, \frac{dX}{dz} - \frac{dZ}{dx}, \frac{dY}{dx} - \frac{dX}{dy}$$

where i,j,k are unity.

Now, in order to get the dynamical equations it has to develop in the state of variation

$$\delta \int (K - P) dt = 0$$

* Elements of natural philosophy, by Lord Kelvin and Tait.

The deduction of this equation gives the following relation

$$\frac{1}{4\pi} \left(\frac{dh}{dy} - \frac{dg}{dz}, \frac{df}{dz} - \frac{dh}{dx}, \frac{dg}{dx} - \frac{df}{dy} \right) = -I(\dot{x}, \dot{y}, \dot{z})$$

which is identical with the Maxwell's solenoidal form.

$$c \text{ Curl } \dot{\mathbf{d}} = -\dot{\mathbf{j}}$$

where c = velocity of light, $\dot{\mathbf{d}}$ = electric force in free aether and $\dot{\mathbf{j}}$ = magnetic force in the free aether.

Taking electrons into consideration together with the aether the flux of the total electric displacement is the sum of the flux of the aethereal displacement due to the change of position of electrons and the flux of the true electric changes; in other words, the rate of change of the aethereal displacement $(\dot{f}, \dot{g}, \dot{h}) d\tau$ in the element of the volume $d\tau$ must be added $\sum(e\dot{x}, e\dot{y}, e\dot{z})$, where $(\dot{x}, \dot{y}, \dot{z})$ is the velocity of an electron, carrying charge e . The total current is, therefore, made up of the drift of electrons and the time rate of change in electric displacement of the aether. This current of the total electrical displacement is itself solenoidal (see appendix A).

In transposing the kinetic energy equation

$$K = 1/2 I \int (\dot{x}^2 + \dot{y}^2 + \dot{z}^2) d\tau \quad \dots \dots \dots (3)$$

into a form which expresses the effect of the motion of the electrons, it is convenient to employ Maxwell's auxiliary vector (F, G, H) having such a relation as

$$\left(\frac{\partial H}{\partial y} - \frac{\partial G}{\partial z}, \frac{\partial F}{\partial z} - \frac{\partial H}{\partial x}, \frac{\partial G}{\partial x} - \frac{\partial F}{\partial y} \right) = (\dot{X}, \dot{Y}, \dot{Z}) \dots (4)$$

Then, we have

$$K = 1/2 I \int (n\dot{Y} - m\dot{Z})F + (l\dot{Z} - n\dot{X})G + (m\dot{X} - l\dot{Y})H \Big] dS + 2\pi I \int (Fu + Gv + Hw) d\tau \dots \dots \dots (5)$$

in which (u, v, w) is the total current and is equal to $(f + u_0,$

$g + v_0, h + w_0), (u_0, v_0, w_0)$ being the true current due to the electrons

It has evidently a relation

$$\text{Curl}(\dot{X}, \dot{Y}, \dot{Z}) = 4(u, w, v) \quad \dots \dots \dots (6)$$

By the equations (4) and (6), we will have a solution for each u, v , and w in the type of:

$$-4\pi u = \frac{d^2 F}{dz^2} - \frac{d^2 F}{dy^2} - \frac{d^2 G}{dx dy} - \frac{d^2 H}{dx dz}$$

$$-4\pi v = \frac{d^2 G}{dx^2} \text{ etc.}$$

$$-4\pi w = \frac{d^2 H}{dy^2} \text{ etc.}$$

The solution of these differential equations can be concrete into equation

$$(F, G, H) = 1/r(u, v, w) d$$

r is the ~~vector~~^{distance} from the point (u, v, w) at which (F, G, H) is measured.

Substituting the values back into the equation (5) in which the surface integral becomes zero because of the continuity of (F, G, H) , it gives

$$K = 2\pi I \iint (u_1 u_2 + v_1 v_2 + w_1 w_2) 1/r_{12} d\tau_1 d\tau_2$$

In order to develop this the dynamical equation of variation with respect to both independent variables, viz, the drift of electrons and displacement of the aether, it is necessary to introduce Lagrangian undetermined function* of position, like Larmor and others, *adopted*.

Then the variation becomes

$$\frac{1}{4\pi I} \int (K - P) dt + \int dt \left[\int \psi \left(\frac{df}{dx} + \frac{dg}{dy} + \frac{dh}{dz} \right) d\tau - \sum e \psi \right] = 0 \quad \dots (7)$$

From (5) we have

* See Maxwell's Electricity and Magnetism, vol. II.

$$\frac{1}{4\pi I} K = 1/2 N e^2 (\dot{x}^2 + \dot{y}^2 + \dot{z}^2) + e(\dot{x}F + \dot{y}G + \dot{z}H) \dots (8)$$

where $(\dot{x}, \dot{y}, \dot{z})$ is the velocity of electron e ; N is a constant which is determinable from experiment; $N e^2$ is effective mass of an electron.

The variation is then

$$\begin{aligned} \frac{1}{4\pi I} \delta K dt &= \int dt N e^2 (\dot{x} \delta \dot{x} + \dot{y} \delta \dot{y} + \dot{z} \delta \dot{z}) \\ &+ \int dt \left\{ e \dot{x} \left(\frac{dF}{dx} \dot{x} + \frac{dF}{dy} \dot{y} + \frac{dF}{dz} \dot{z} \right) + e \dot{y} \left(\frac{dG}{dx} \dot{x} + \dots \right) + e \dot{z} \left(\dots \right) \right\} \\ &+ \int dt (eF \delta \dot{x} + eG \delta \dot{y} + eH \delta \dot{z}) \end{aligned}$$

Referring to the time integral of traveling electron the equation will be in the following form;

$$\begin{aligned} \frac{1}{4\pi I} \delta K dt &= \int dt \delta x \left\{ - N e^2 \ddot{x} + e \left\{ \dot{y} \left(\frac{dG}{dx} - \frac{dF}{dy} \right) - \dot{z} \left(\frac{dF}{dz} - \frac{dH}{dx} \right) - \frac{dF}{dt} \right\} \right\} \\ &+ \int dt \delta y \left[\dots \right] + \int dt \delta z \left[\dots \right] . \end{aligned}$$

The last half of the first term becomes

$$\begin{aligned} &\delta \int dt \int \psi \left(\frac{df}{dx} + \frac{dg}{dy} + \frac{dh}{dz} \right) d\tau \\ &= \int dt \int \psi (l \delta f + m \delta g + n \delta h) dS \\ &- \int dt \int \left(\frac{d\psi}{dx} \delta f + \frac{d\psi}{dy} \delta g + \frac{d\psi}{dz} \delta h \right) d\tau . \end{aligned}$$

The last term

$$\delta \int dt \sum e \psi = \int dt \sum e \left(\frac{d\psi}{dx} \delta x + \frac{d\psi}{dy} \delta y + \frac{d\psi}{dz} \delta z \right)$$

The sum of all these four terms is zero. Hence the coefficient of each group must also be zero. Gathering all terms together, taking x component only here, we have

$$\int dt \delta x \left\{ - N e^2 \ddot{x} + e \left\{ \dot{y} \left(\frac{dG}{dx} - \frac{dF}{dy} \right) - \dot{z} \left(\frac{dF}{dz} - \frac{dH}{dx} \right) - \frac{dF}{dt} \right\} \right\}$$

$$- \frac{M}{4\pi I} \int dt \int f \delta f d\tau + \int dt \int \psi (\mathcal{L} \delta f) dS - \int dt \int \left(\frac{d\psi}{dx} \delta f \right) d\tau - \int dt e \left(\frac{d\psi}{dx} \delta x \right)$$

$$\text{or } \left[-Ne^2 \ddot{x} + e(\dot{y}\dot{z} - \dot{z}\dot{y} - \frac{dF}{dt} - \frac{d\psi}{dx}) \right] \int dt \delta x + \left[-\frac{M}{4\pi I} - \frac{d\psi}{dx} + \psi \mathcal{L} \right] \int dt \int \delta f.$$

since $\psi \mathcal{L} = -\frac{d\psi}{dx}$, from which

$$Ne^2 \ddot{x} + e(\dot{y}\dot{z} - \dot{z}\dot{y} - \frac{dF}{dt} - \frac{d\psi}{dx}) = 0$$

$$-\frac{M}{4\pi I} - \frac{d\psi}{dx} - \frac{d\psi}{dx} = 0$$

writing $(4\pi c)^2$ for M/I .

$$\text{Then } 4\pi c^2 f = -\frac{dF}{dt} - \frac{d\psi}{dx} \dots \dots \dots (1)$$

$$Ne^2 \ddot{x} = e(\dot{y}\dot{z} - \dot{z}\dot{y} - \frac{dF}{dt} - \frac{d\psi}{dx}) \dots \dots \dots (2)$$

The right hand side of the equation (1) is the x component of the aethereal force and that of equation (2) is the x component of the true electric force which tends to accelerate the motion of an electron e.

Since a current of conduction is made up of electrons and positive ions travelling in opposite direction under an impressed electric force, no matter how great the relative velocities of the two, the last two terms of the equation (2) annuls, and the rest terms give a force of, now considering the three components, $(\dot{z}\Sigma e\dot{y} - \dot{y}\Sigma e\dot{z}, \dot{x}\Sigma e\dot{z} - \dot{z}\Sigma e\dot{x}, \dot{y}\Sigma e\dot{x} - \dot{x}\Sigma e\dot{y})$.

If (u_0, v_0, w_0) represents the true current per unit volume, from definition

$$(\Sigma e\dot{x}, \Sigma e\dot{y}, \Sigma e\dot{z}) = (u_0 v_0 w_0) d\tau$$

Hence this force is $(v_0 z - w_0 y, w_0 x - u_0 z, u_0 y - u_0 x)$ per

unit volume of material medium. This is an expression of the mechanical force acting on a conducting body carrying a current.

VII. Equations directly connected to the Electron theory.

In the following, some other important expressions which are deduced from the fundamental theories and the experimental evidences are treated. These are closely connected to the phenomena of the general discharges as well as arc discharge.

As a current is consisted of the drift of electrons, each having a negative electricity e , travelling through a medium in one direction, viz., from negative to positive direction, and positive ions travelling in the opposite direction, it is generally expressed as

$$i = N e v + N_1 e v_1 \quad (1')$$

where N and N_1 represent number of electrons and positive ions in a unit volume, e is the charge on each electron or ion, v and v_1 are the average velocities of the translation of electron and ion through the material of the medium. Due to the very small velocity of ion compared to that of electron, the second term of the right hand side of the equation can be neglected. Thus

$$i = N e v \quad (1)$$

The electrons move freely through conductors and it is

supposed that they behave like a perfect gas; consequently they are considered to obey the kinetic theory of gases. In simplifying the problem, in place of complexed Maxwell-Boltzmann's form, an assumption is made that all electrons are moving with same velocity. Then, from the kinetic theory of gases, the velocity and the temperature of the conductor are connected as

$$\frac{1}{2} m u^2 = a \theta \quad \dots \dots \dots (2)$$

where m represents the mass of an electron,

u represents mean velocity of electron,

θ represents the absolute temperature

and a represents constant for any substance.

It is also evident that when an electron of mass m carrying a charge e is placed in an electric field of intensity X , it will be subject to acceleration, g

$$g = X \cdot e/m \quad \dots \dots \dots (3)$$

The distance which will be passed in time t is of course

$$s = \frac{1}{2} g t^2 = \frac{1}{2} X \cdot e/m t^2 \quad \dots \dots \dots (4)$$

where the time t is the duration of two collisions between the electron and the molecules. Hence, if ℓ represents the distance travelled by the electron between two collisions and u the mean velocity of the electron, time will be

$$t = \ell/u \quad \dots \dots \dots (5)$$

Substituting this value into the equation (4) we have

$$s = \frac{1}{2} X \cdot e/m (\ell/u)^2 \quad \dots \dots \dots (6)$$

Since v , as before, is the velocity of the translation of electron through the medium and that is the distance passed in unit time, must be s/t ,

$$v = s/t = \frac{1}{2} X \cdot e/m \ell/u \quad \dots \dots \dots (7)$$

But, from equation (2)

$$u = (2a\theta/m)^{1/2}$$

Substituting this value in equation (7)

$$v = \frac{X e \ell}{2^{1/2} (a m \theta)^{1/2}} \dots \dots \dots (8)$$

From this value the current equation (1) can be expressed in terms of the electric intensity and absolute temperature:

$$i = e v N = \frac{X e^2 \ell N}{2^{3/2} (a m \theta)^{1/2}} \dots \dots \dots (9)$$

where e , ℓ , N , a , m are all constants for a particular conductor and therefore if θ is kept constant i is proportional to X and Ohm's law holds well. From this equation it is also seen that i is inversely proportional to the sq. root of the temperature.

The kinetic energy due to the motion of an electron can be derived by the aid of the preceding equations.

The total kinetic energy of electrons can, as it is seen in previous theory, section (VI), be considered as made up of two parts, the energy due to mass of electrons themselves and that due to the displacement current of surrounding aether; thus

$$K. E. = 1/2(m+m')v^2 \dots \dots \dots (10)$$

where $1/2 m' v^2$ is the energy due to the latter. J.H. Jeans gives the value for this quantity as $\frac{e^2}{3 a c^2} v^2$, where c is the velocity of light, a is radius of the sphere on which the electric charge e is uniformly distributed. The formula (10) is true only when the velocity v remains constant.

Substituting the value of v from (8) in the equation (10)

$$K.E. = \frac{X^2 e^2 \ell^2 (m+m')}{16 a m \theta} \dots \dots \dots (11)$$

N.R. Campbell* gives an equation for the energy in the form

* Morder theory of electricity: page 225.

$$\text{K.E.} = \frac{1}{2} \frac{x^2 e^2 l^2}{m u^2}$$

which is directly derived out from the equation (7) .

An expression for the potential difference between electrode plates

The Poisson's equation gives

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = -4\pi\rho$$

In this case, however, the electrode plates are parallel and the tubes of force are perpendicular to the plates. Then we may write

$$\frac{\partial^2 V}{\partial x^2} = 4\pi\rho$$

where V equal potential at a distance x from a terminal, ρ is the volume density of the electricity. Since electric intensity, X, is dV/dx , the last expression can be written

$$\frac{\partial}{\partial x} \left(\frac{dV}{dx} \right) = \left(\frac{dX}{dx} \right) = 4\pi\rho$$

Supposing that the density ρ is due to the electrons only, ρ must equal to $N e$ in which N represents the number of electrons per unit volume.

Hence, it follows

$$\frac{dX}{dx} = 4\pi N e \dots\dots\dots (12)$$

From equation (9)

$$X = \frac{2^{3/2} (a_m \theta)^{1/2}}{e^2 l N}$$

Mutiplied together we have

$$X = \frac{dX}{dx} = \frac{2^{3/2} (a_m \theta)^{1/2} 4\pi}{e l} \dots\dots\dots (13)$$

which is equivalent to J.J. Thomson's equation :

$$X \frac{dX}{dx} = \frac{4\pi i}{v_1}$$

v_1 is the velocity of the electron under unit electric field. For simplicity his equation is taken. Integrating it we have:

$$X^2 = \frac{8\pi i x}{v_1} + c \dots \dots \dots (14)$$

c is the the integration constant. Thus :

$$X = \left(\frac{8\pi i x}{v_1} + c \right)^{1/2}$$

Since the potential difference between any two points is $X dx$, the potential difference between cathode and a point a small distance s from the terminal will be :

$$\begin{aligned} V_s &= \int_0^s X dx = \int_0^s \left(\frac{8\pi i x}{v_1} + c \right)^{1/2} dx \\ &= \frac{2}{24} \frac{v_1}{i} \left(\frac{8\pi i s}{v_1} + c \right)^{3/2} - \frac{2}{24} \frac{v_1}{i} c^{3/2} \dots \dots \dots (15) \end{aligned}$$

According to J.J. Thomson*, the integration constant c may be neglected. Thus the equation becomes approximately .

$$\begin{aligned} V_s^2 &= \left(\frac{v}{12\pi i} \right)^2 \left(\frac{8\pi i s}{v_1} \right)^3 = \frac{32\pi i s^3}{9 v_1} \\ V_s &= \sqrt{\frac{32\pi i s^3}{9 v_1}} \dots \dots \dots (16) \end{aligned}$$

while J.J. Thomson's equation is

$$V_s^2 = \frac{32\pi i \ell^3}{9 v_1}$$

where ℓ is the distance between plates.

* Conduction of Electricity through Gases: second edition p,101.

It is seen, from this expression, that Ohm's law will no longer hold good in the case where electricity passes through gas. In the curves of the No. 12 in my experiment, the nature will be seen. Owing to the following reason, however, i does not change just as in the expression (16).

In treating this subject, J.J. Thomson supposed first that the current is carried entirely by ions of one sign, secondly that the electric intensity is considered to be a constant at any point throughout the space between the electrodes. This is not always true, especially for an arc discharge, because from point to point between the electrodes, n , v , and, consequently, X are changing due to the acceleration or deceleration of ion and recombination of positive and negative ions.

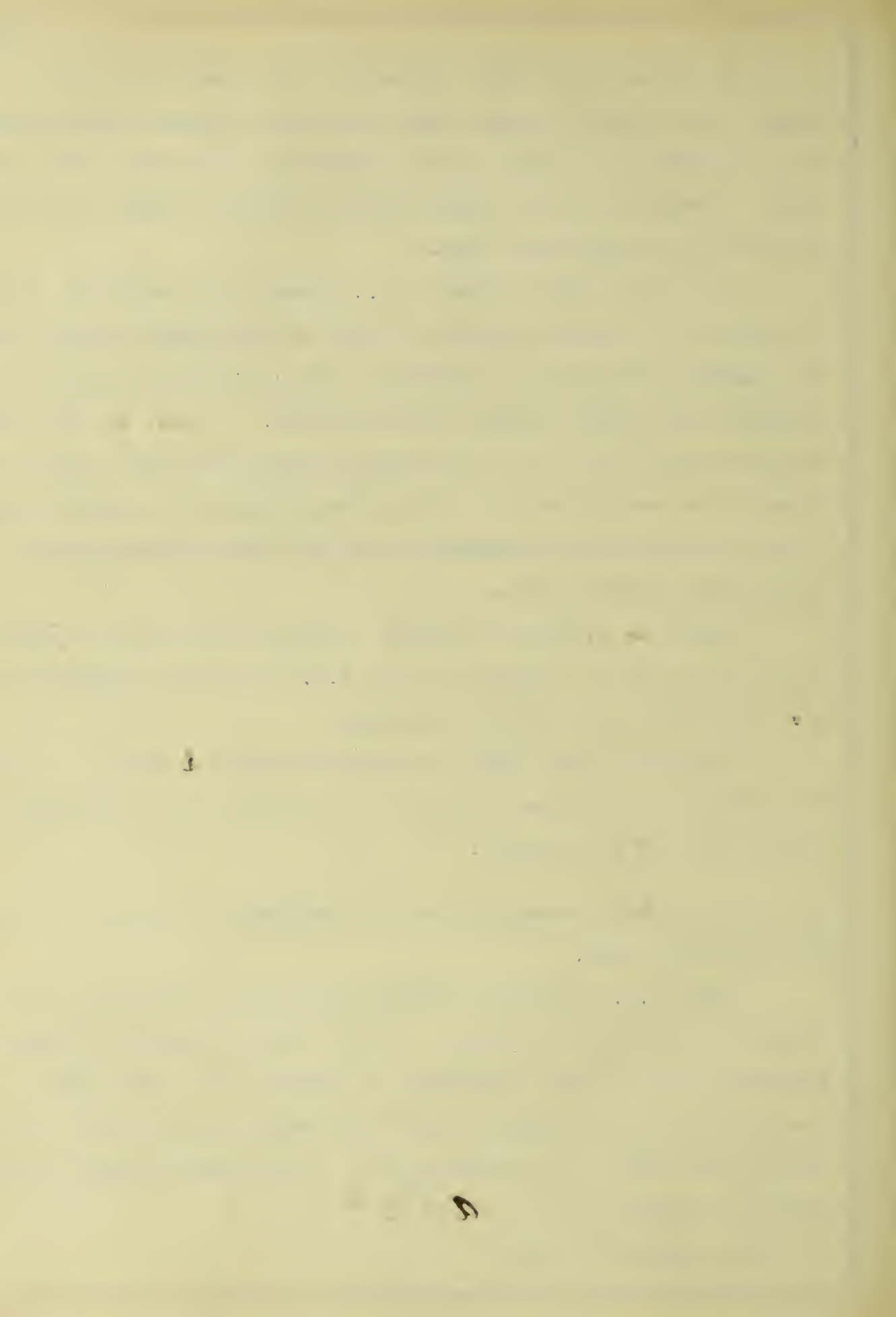
Hence, by dividing the space between electrodes into n number of sections and the summation of the P.D.s of all the sections will give the total P.D. between electrodes.

His formula then must be interpreted that i does not extend from terminal to terminal but only for the space of a particular section where X is constant.

The effect of high temperature on the ionisation produced by carbon and metals into gas.

Prof. O.W. Richardson* made a systematic measurement of the ionisation produced by carbon. He is considering that the current produced is due to the liberation of electrons from the carbon. His ionisation current ranges from 10^{-8} to 2 amperes per square cm. of carbon according to the temperature. The general formula deduced by him is written as $I = a\theta^{\frac{1}{2}}e^{-\frac{b}{\theta}}$

* Phil. Magazine: 1903.



where a and b are constant, θ is the absolute temperature.

This formula has to be modified when the forces to which the electrons are subject become appreciable.****

F. Deininger* confirmed Richardson's formula by repeating his experiments on the same subject using various metals and carbon.

H. A. Wilson** investigated and found that it varies with the previous treatment of metals. He agreed with Richardson's results with carbon.

J.N. Pring and A. Parker*** made experiment on this, and give a marked difference in their results giving very small value for I compared ^{with} ~~that~~ from Richardson's formula, although they do not derive any definite formula for the correction.

The values for the constants found by several experimenters are:

substance	a	b	experimenter
carbon	10^{34}	$9.8 \cdot 10^4$	Richardson
platinum	$1.5 \cdot 10^{26}$	$4.93 \cdot 10^4$	"
"	$6 \cdot 10^{25}$	$7.75 \cdot 10^4$	H.A. Wilson
diff. treat.	$6.9 \cdot 10^{26}$	$6.55 \cdot 10^4$	"

The relation between the negative ionisation or "negative leak" from ^ahot body and the temperature is still in question, notwithstanding the ~~for~~^egoing Richardson's formula, on which many experimenters have widely different views, and thus it is not completely settled.

On the "positive leak" or the escaping ^{of} the positive electricities from the hot body, as it is more complex than the negative

* Der Deutsch Physik Gesell; 1907

** Phil. Trans.: 1904

*** Phil. Mag. June, 1912.

**** "Electron theory" by Richardson; Phil. Mag. Nov. 1912.

leak, there is little information, in spite of the recent works of Richardson and some other experimenters'. Pring and Parker* have found no trace of the positive leak from carbon, up to 2000°C when the field was excited 200 volts.

* Phil. Magazine; Jan., 1912.

VIII. Electron theory of an Electric Arc.

The general description of the arc is given in the first part of this paper. Here it is proposed to investigate, from the point of view of the electron theory, some important question of the relation between the resistance and the potential drops which arise from a consideration of Mr. Duddell's research*. It is also attempted to criticize the explanation by Prof. T.A. Pollock** on the same subject. Mr. Duddell used solid carbons for electrodes and gave the following results:

At the anode surface a back e.m.f. of 16.7 volts followed by a fall of 16 volts in a very short distance from the terminal, then a fall of 25 volts through the vapor body, 11 volts in a short distance from the cathode and 6.1 volts at the surface of it.

Prof. Pollock tried to show the phenomena under the following principle: "If the hot carbon is the cathode, so that the electrons are projected in the direction in which negative ions are being carried by the field the surface P.D. is a forward e.m.f. helping the flow of current,-----if the carbon is anode, so that the electrons are projecting in the direction opposite to that in which negative ions are being carried by the field, the surface P.D. is in back e.m.f. opposing the flow of current".

He supposed that the enormous number of electrons and positive ions are projecting from the surface of carbon no matter whether the carbon is negative or positive. For the purpose of

* Phil. Trans: 1904.

** Phil. Mag. March, 1909

general description the emissions of positive ions is neglected due to their small velocities. His explanation ^{is} based upon ^{the} above assumption. Unfortunately, however, he is considering, as it seems to me, that the electrons and positive ions are projecting from the anode against the applied field which is evidently contradictory to many experimental facts. Fleming* from his experiment suggests that a current can pass in the direction in which the negative electricity comes from the hot electrode into the gas but not in the other direction. Pring and Parkers** also give a contrary result. The reason is hereafter given and proved.

The phenomena of Arc.

As the terminals are in contact, the electrons pass through the conductor ~~with~~ a certain velocity under the applied field. But when the terminals are separated, the electrons are accumulated at the point of separation of the cathode terminal ^{because of} the much greater coefficient of the electronic resistivity of gas than that of ^{the} electrode; this depends upon the constituents of molecules.

The kinetic energy of electrons due to the change of velocities is converted into heat, the amount of which is, neglecting that energy of the displacement current in the aether;

$$\frac{1}{2} M(v'^2 - v^2) \frac{1}{T} = J H.$$

This is work done by mass M due to the change of velocities from v to v' in any time T in ergs. J is the mechanical equivalent of heat which is 4.19×10^7 and H is the heat measured in calories.

Then a part of ^{the} electrons shoot out from the cathode under the influence of heat and impressed e.m.f. which travel to the

* Proc. Roy. Soc. 1890.

** Phil. Mag. Jan., 1912.

anode where the recombination take place with positive ions which in turn generates heat. The ionisation takes place due to the heat and impressed e.m.f. at both the anode terminal and the gas surrounding the terminal. A part of positive ions of the electrode ^{are} liberated from there and migrate towards the cathode where they recombine with the accumulated electrons, the ~~rest~~ ^{remaining} part of which projects out under the heat and stress of the applied e.m.f. Thus a certain number of electrons travels from cathode to anode while the positive ions migrate in the opposite direction. This is a simple statement of the mode of conduction in gas.

Recombination theory.

Since the already existing electrons or ions can not be nullified due to the conservation of system they would infinitely increase in number at the ionising sections, consequently infinite increase of the current in the circuit.

This is contrary to the ordinary case and ~~which~~ is in fact checked by what here called recombination.

The Recombination is the process in which electrons and ions unite together forming atoms in neutralising state. These neutralized atoms deposit ^a at the place where the action took place unless they are tossed away by some other means.

Recombination occurs where the velocities of electrons and ions are changing and must be ~~where~~ ^{are} positive ions ^{are} present. The result of the recombination is the generation of heat due to their kinetic energy. The amount is

$$\frac{1}{2} m v_1^2 + \frac{1}{2} M v_2^2 = J H$$

This is the action called by some writers the bombardment of the electrode terminals by ions or electrons.

Theoretically, therefore, if the molecules of a conductor are arranged so uniformly or regularly as not to cause the change of velocities of ions and electrons, there would be no resistance drop in the conductor, hence there will be no heat generated.

From this point of view of the electron theory the resistance of a conductor means the recombination of ions and electrons, neutralising and generating heat.

Ionisation theory.

The essential features of the recombination is the presence of ions and electrons and the change of their velocities.

The Ionisation, on the contrary, is a process which acts just exactly opposite to it.

The ionisation needs heat, but heat only can not ionize a substance if there is no electronic assistance . Dr. N. R. Campbell* says, in his book; " It is clear that a molecule can not acquire sufficient energy at any attainable temperature to ionize the atom of a gas by collision with it. For energy required to ionize the atom of a gas is 4.2×10^{-11} ergs. The kinetic energy of a molecule at the temperature T is aT where $a = 1.5 \times 10^{-16}$. Therefore $T = 4.2 \times 10^{-11} / 1.5 \times 10^{-16} = 2.8 \times 10^5$ where as it ^{is} not our power to produce a temperature of more than 4×10^3 ."

The heat simply helps the ionising action by giving ions and electrons more agitation.

Gas ions and Potential Difference .

The positive ions produced by the ionisation of the gas at the anode is entirely different from those coming from the electrode itself. A part of those ions tends to draw back electrons for recombination. This is the very cause of back e.m.f. at the anode

end.

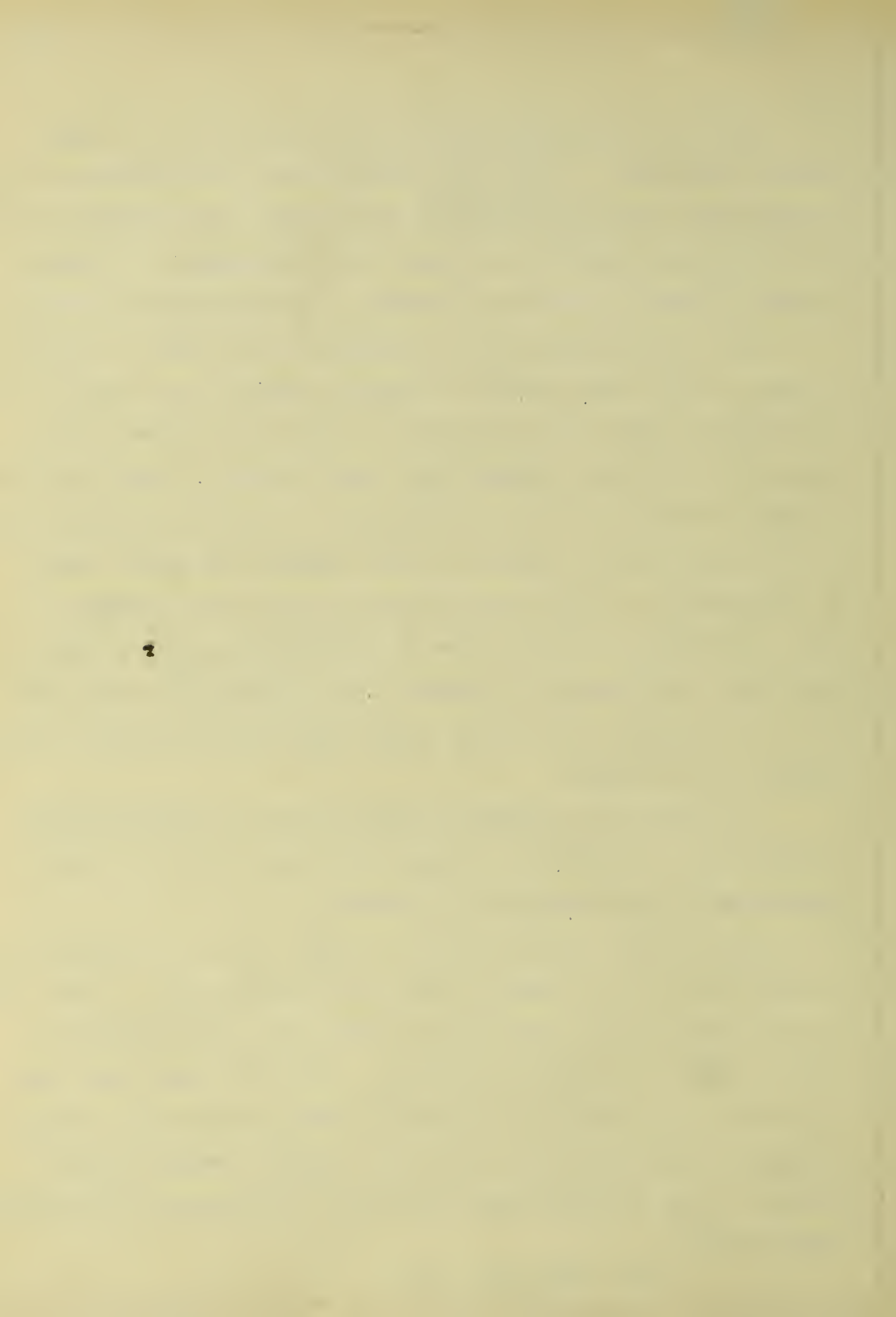
The rest of the gas ions are all recombined in the space between electrodes. Thus the ionisation and recombination of the gas ~~is~~ always balanced ^{each other,} ~~themselves,~~ hence there is no positive ions of the gas going into the electrode unless the nature of ions is the same as that of positive electrode as in electrolytic process.

Sir Oliver Lodge* in his discussion on the mode of conduction in gases, states that however plentifully electrons are supplied to the cathode by the source of current, they do not spontaneously escape from the metal, except under some stimulus, unless positively charged ions are in their ^{gn} neighbourhood---positively charged ions of gas electrified by contact with the positive electrode travel to the cathode.etc. And he conclude that it is at the surface of anode, therefore, the separation of electricities really takes place under the stress of applied e.m.f. and the ions then migrate to the cathode, from which extract an electron apiece, thereby becoming neutral again.

He regards the uncharged atoms which are diffused towards the anode and the positively charged ions away from it. are the essential~~ly~~ to the passage of the current.

These views, unfortunately, it seems to me, are contradictory, to some extent, to the modern views of electron^{ci} theory. There is no such theory as that particles of a gas will become charged by contact with an electrified surface. Also it is well known and recognized by the majority of physicists that no positively charged particles are emitted from a positively charged terminal without presence of any electrons which cause the recombination at ordinary temperature.

* Phil. Mag. July, 1911.



On the "Electric theory of an Electric Arc".

The writer is very glad to have had the able comments on this paper by Dr. J. Kunz of the Department of Physics, and he highly appreciates his suggestive explanations.

Dr. Kunz's view, however, differs from the writer's in some respect with regard to the arc phenomena. Notwithstanding Dr. Kunz's explanations, the writer can not all agree with them. He is really willing to adopt any view if it is satisfactorily proved to be reasonable or true. Unfortunately, here in this case, the writer can hardly find any sound reason in Dr. Kunz's view which can possibly ~~be~~ displace his own reasoning.

Due to the lack of time, however the writer is not able to disprove by experiments or by other means.

From these reasons, the writer's electronic theory of the arc is retained as it is. The arc phenomenon is due to the drift of electrons travelling from the cathode to the anode together with the positive ions from the gas, and the positive electrode moving in the opposite direction. In this view we agree.

When separating the electrodes the high temperature at the ends of the terminals are due to resistance of the very minute contact, this is the same view as ~~was~~ held by Mrs. Ayrton's, twenty years ago, and this explanation has no bearing on the electronic theory.

Dr. Kunz's view of the relative velocities of electrons in metal and in gas is brought forth in the following.

The number of electrons in metal is very much greater than that in gas and the velocity in the metal is much slower than in gas.

The current in the metal is carried merely by the drift of electrons while that in the gas is carried by both the electrons and positive ions.

From Dr. Kunz's point of view the relative velocities of the electrons may be derived by the aid of current equation; $i = nev$.

$$\text{In metal} \quad i = n_{m-} e v_{m-} \dots\dots\dots (a)$$

$$\text{In gas} \quad i = n_{a-} e v_{a-} + n_{a+} e v_{a+} \dots\dots\dots (b)$$

Taking the most simple case, where the ionization between the electrodes is entirely disregarded. Then, the number of electrons in metal must be equal to the sum of the numbers of electrons and positive ions in the gas because the same current is flowing through the circuit. Or

$$n_{m-} = n_{a-} + n_{a+} \dots\dots\dots (1)$$

With the same heat agitation, the ratio of the velocities of electron and positive ions is roughly

$$\begin{aligned} \frac{v_{a-}}{v_{a+}} &= 65 & [1/2 m v^2 = \alpha \theta] \\ v_{a+} &= \frac{v_{a-}}{65} \dots\dots\dots (2) \end{aligned}$$

Substituting the value of (1) into (a) and the value of (2) into (b)

$$i = (n_{a-} + n_{a+}) e v_{m-} \dots\dots\dots (3)$$

$$\begin{aligned} i &= n_{a-} e v_{a-} + n_{a+} e v_{a-}/65 \\ &= e v_{a-} (n_{a-} + n_{a+}/65) \dots\dots\dots (4) \end{aligned}$$

Equating (3) and (4)

$$\begin{aligned} (n_{a-} + n_{a+}) e v_{m-} &= (n_{a-} + n_{a+}/65) e v_{a-} \\ \text{or} \quad \frac{v_{m-}}{v_{a-}} &= \frac{n_{a-} + n_{a+}/65}{n_{a-} + n_{a+}} \end{aligned}$$

This is an expression of the relative velocities of elec-

trons in metal and in gas, in terms of the numbers of electrons and positive ions in the gas without regard to the ionisation. This ratio, therefore, depends on the number of positive ions and their velocities.

As the matter of fact, however, ionisation take place throughout the arc, the number of electrons and positive ions increase very greatly when equation (1) does not hold. Hence, on this account alone, the velocity of electron in metal may be greater than in gas.

Moreover, it seems not reasonable to suppose that the velocity of electron in a conductor is less than that in an insulator. The idea seems antagonistic to the well recognized principle that electron travel about freely through conductor, but not through insulators.

The high temperatures at the cathode and anode are due to the bombardments by the positive ions and electrons respectively. This essentially agrees with the writer's idea of the recombination process at the electrodes, though they are differently expressed.

Dr. Kunz's view, furthermore, does not explain the cause of the back e.m.f. at the anode of a carbon arc, where as the theory proposed by the writer appears to explain this phenomenon in a satisfactory way.

IX. Appendix A.

If there are electrons in medium, by Faraday's hypothesis

$$\int (l f + m g + n h) dS = \sum e$$

where l, m, n are the direction cosines.

Therefore in any finite charge Δ

$$\Delta \int (l f + m g + n h) dS$$

is equal to the flux of electrons into the region across the boundary. Then

$$\frac{d}{dt} \int (l f + m g + n h) dS = - \int (l u_0 + m v_0 + n w_0) dS$$

where (u_0, v_0, w_0) is the true electric current which is simply the flux of electrons reckoned per unit time. Transposing and combining, we have for any closed surface

$$\int (l u + m v + n w) dS = 0$$

in which $(u, v, w) = (df/dt + u_0, dg/dt + v_0, dh/dt + w_0)$

Thus the divergence is zero, the resulting current is solenoidal.



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